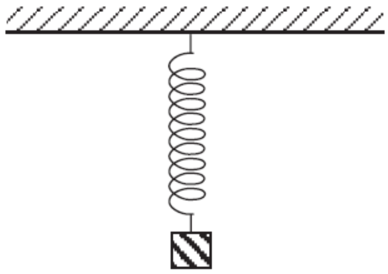
Hooke's Law & SHM

2005 SP

7. Large springs for shock absorbers are tested at a research laboratory. A body of mass 50 kg is suspended from a test spring of natural length 0·80 metres, which has the other end attached to a fixed horizontal surface.



(*a*) Given that, when in equilibrium, the body extends the spring by 0·14 metres, find the modulus of elasticity of the spring. **2**

(*b*) The body is now pulled 0·20 metres vertically down from its equilibrium position and then released from rest. Take y metres as the vertical displacement of the body from its equilibrium position, t seconds after release. Show that, when all resistive forces are ignored,

$$\frac{d^2y}{dt^2} + 70y = 0.$$ **3**

2010

A6. A toy car of mass 250 grams is stationary on a smooth horizontal surface. One end of a light spring is attached to the car, the other end is fixed to the surface. The natural length of the spring is 1 metre and the modulus of elasticity is 4 newtons.

The car is pulled along the surface, extending the spring by 20 centimetres, and then released.

(*a*) Show that the displacement, x metres, of the car from its equilibrium position satisfies an equation of the form

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where the value of the constant ω should be stated. **3**

(*b*) Calculate the maximum speed of the car. **2**

2013

A7. A light elastic string of natural length l metres hangs from a fixed point O with a particle of mass m kilograms attached at its lower end. In equilibrium the string is extended by e metres.

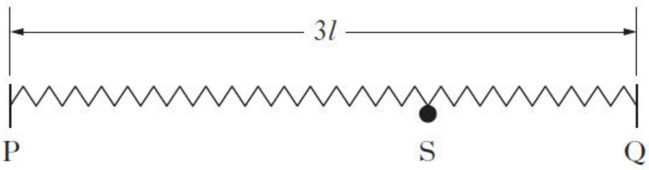
The particle is then pulled down a further distance a metres where $a < e$ and released.

Show that the ensuing motion is simple harmonic and state the period of the motion. **4**

The maximum velocity of the particle during motion is $\frac{1}{2}\sqrt{ge}$. Find an expression for the amplitude of the motion in terms of e. **2**

2015

A10. Two light elastic springs, each of natural length l metres are attached to a particle S of mass m kg. The particle initially lies in equilibrium on a smooth horizontal table, with the springs attached to two fixed points P and Q, a distance of $3l$ metres apart as shown in the diagram.



The spring attached at P has modulus of elasticity mg newtons and the spring attached at Q has modulus of elasticity $3\,mg$ newtons.

(*a*) Show that, in equilibrium, the distance PS $= \frac{7}{4}l$. **4**

S is then moved along the line PQ so that PS $= \frac{3}{4}l$ and is then released from rest at this point.

(*b*) (i) By finding in terms of l the extensions in PS and QS when the particle is x metres from its equilibrium position, show that the particle subsequently moves with simple harmonic motion. **4**

(ii) Show that the maximum velocity of the particle can be written in the form $k\sqrt{gl}$ and state the value of k. **2**

2017

12. A body of mass 750 grams is attached to a light elastic string of natural length 50 cm and modulus of elasticity 150 N. The mass hangs vertically with one end of the string attached to the ceiling.

(a) Find the extension in the string when the body hangs in equilibrium. **2**

The body is released from a position 2 cm below the equilibrium position.

(b) (i) Show that the body moves with simple harmonic motion modelled by $\ddot{x} = -400x$ where x metres is the displacement from the equilibrium position. **3**

(ii) Find the speed of the body when it is 0·5 cm above the point of release. **2**

(c) On another occasion the body is pulled down 3 cm below the equilibrium position. Explain why, in this case, the subsequent motion is not simple harmonic. **1**

2016 EX

15. A light elastic string of natural length ℓ metres hangs from a fixed point O with a particle of mass m kilograms attached at its lower end. In equilibrium the string is extended by e metres.

The particle is then pulled down a further distance a metres where $a < e$ and released.

(a) Show that the ensuing motion is simple harmonic and state the period of the motion. **4**

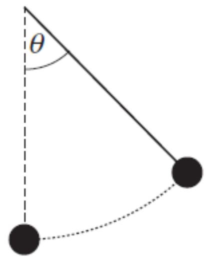
The maximum velocity of the particle during motion is $\frac{1}{2}\sqrt{ge}$.

(b) Find an expression for the amplitude of the motion in terms of e. **2**

Pendulums and SHM

2012

A10. (*a*) A simple pendulum consists of a mass m kg suspended from a fixed point by a light, inextensible string of length L metres. The mass is pulled to the side so that the taut string makes an angle θ with the downwards vertical through the fixed point, as shown in the diagram. The mass is then released from rest.



By considering the forces acting on the mass along the tangent to the circle that the mass describes, show that, for small values of θ,

$$\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta .$$ **3**

Assuming that $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$, find an expression, in terms of L and g, for the period of oscillation of the pendulum and calculate the length of string required for the period to be 2 seconds. **3**

(*b*) A particle moving in a straight line, whose motion is also simple harmonic, oscillates with period 2 seconds about a point O. The particle is moving towards O with speed $4\pi \text{ m s}^{-1}$ when it passes through a point P which is 3 metres from O.

Show that the amplitude of the motion is 5 metres. **2**

Calculate the time which elapses from the instant when the particle leaves P to when it next passes through O. **2**